ILP Formulations for the Lazy Bureaucrat Problem

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Outline

Lazy Bureaucrat Problem

The lazy bureaucrat problem (LBP) is a scheduling problem (common arrivals and deadlines) in which a set of jobs has to be scheduled in the most inefficient way!

Definition

The lazy bureaucrat needs to choose a subset of jobs to execute in a single day, in a such a way that:

- no other job fits in his/her working hours
- the total profit of selected jobs (e.g., their duration) is minimized

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Applications of the Lazy Bureaucrat Problem

Practical point of view

- how to distribute the available budget so that: (i) the minimal amount of money is allocated to funding requests and (ii) while having a good excuse that no additional funds can be granted without violating the available budget?

Theoretical point of view

- new interesting insights as the optimization is driven in the opposite direction when compared to their standard (non-lazy) counterparts

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Lazy Bureaucrat Problem – Mathematical definition

Definition (LBP with common arrivals and deadlines)

- We are given:
  - a set of jobs $I = \{1, \dots, n\}$
  - each job $i \in I$ has a duration $w_i \in \mathbb{N}$ and a profit $p_i \in \mathbb{N}$
  - all jobs arrive at the same time
  - all jobs have a common deadline $C \in \mathbb{N}$
- The goal is to find a least profitable subset of jobs $S^* \subset I$ such that:

$$S^* = \arg\min_{S \subset I} \{ \sum_{i \in S} p_i \mid \sum_{i \in S} w_i \leq C \text{ and } \sum_{i \in S} w_i + w_j > C, \; \forall j \notin S \}$$

- `time-spent` objective ($p_i = w_i$).
- `min-number-of-jobs` objective ($p_i = 1$).
- general objective function `weighted-sum`

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Complexity issues

- The problem has been introduced in [1] where it was shown that a more general problem variant with individual arrival times and deadlines is NP-hard
- For the problem variant with common arrival times and deadlines, [3] show that the problem is weakly NP-hard for the `min-number-of-jobs` objective by reduction from subset-sum.
- Thus, the problem studied in this paper (with the more general `weighted-sum` objective) is also at least weakly NP-hard.
- In [5] a FPTAS have been proposed for the `time-spent` objective with common arrival and deadlines.

Property

The `weighted-sum` lazy bureaucrat problem with common arrivals and deadlines is weakly NP-hard

Relationship with the Knapsack Problem

- the LBP can be seen as the problem of packing a set of items in a knapsack in a most inefficient but feasible way
- jobs of I are the items
- job durations w_i are the item weights
- job profits p_i are the item profits
- the deadline C is the budget or capacity.

Notation and preprocessing

- items sorted in non-decreasing lexicographic order:
- non-decreasing weights $w_1 \leq w_2 \leq \cdots \leq w_n$
- non-decreasing profits p_i

- $C_i := C - w_i$, $C_1 \geq C_2 \geq \ldots C_n$.
- $w_{\max} := \max_{i \in I} w_i (= w_n)$
- $w_{\min} := \min_{i \in I} w_i (= w_1)$
- $W := \sum_{i \in I} w_i$
- $P := \sum_{i \in I} p_i$

SOLUTION PROPERTIES

Solution Properties

Definition (Maximal Knapsack Packing)

- A packing $\mathcal{P}$ is a set of items, with the property $\sum_{i \in \mathcal{P}} w_i \leq C$, i.e., a subset of items, which does not exceed the capacity C.
- A packing $\mathcal{P}$ is called maximal, iff $\mathcal{P} \cup \{i\}$ is not a packing for any $i \notin \mathcal{P}$.

Property

Each feasible LBP solution corresponds to a maximal feasible packing of the knapsack with capacity C.

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The capacity used by an arbitrary feasible solution S is bounded from below by $C - w_{\max} + 1$.

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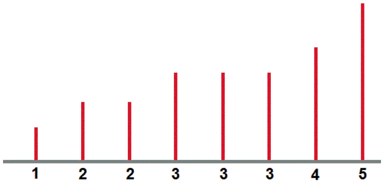
Critical Item (i)

We can only consider the minimum weight item outside the knapsack!

Definition (Critical Weight and Critical Item.)

$$i_c = \arg\min\{i \in I \mid \sum_{j \leq i} w_j > C\}$$

- the index of a critical item is the index of the first item that exceeds the capacity, assuming all $i \leq i_c$ will be taken as well.
- The critical weight (w_c) is the weight of the critical item ($w_c = w_{i_c}$).



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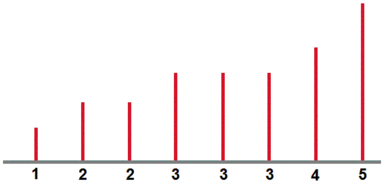

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Critical Item (ii)

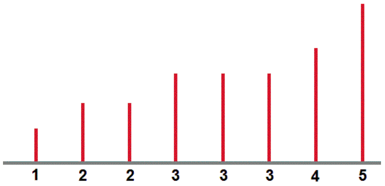
Proposition

The weight of the smallest item left out of any feasible LBP solution is bounded above by the critical weight w_c, i.e.:

$$S \text{ is feasible } \Rightarrow \min_{i \notin S} w_i \leq w_c.$$

Consequently, the size of the knapsack can be bounded from below as:

$$w(S) \geq C - w_c + 1.$$



Example

Weights(=profits) $\{1, 2, 2, 3, 3, 3, 4, 5\}$ and $C = 6$; $i_c = 4$ and $w_c = 3$

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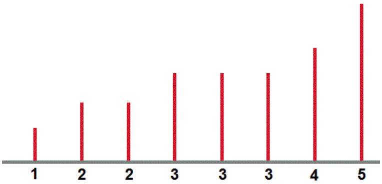

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Main Observation

Observation

Once, the smallest item left out of the solution is known, the problem reduces to solving the knapsack problem with a lower and upper bound on its capacity (LU-KP)

If items are sorted in non-increasing order acc. to w_i and i is the smallest item outside:

$$(\mathrm{KP}_i) \quad J^* = \arg \min_{J \subseteq \{i+1,\dots,n\}} \{\sum_{j \in J} p_j + P_i \mid C - w_i - W_i + 1 \leq \sum_{j \in J} w_j \leq C - W_i\}$$

Based on it:

- FPTAS [5]
- Dynamic Programming ⇒ **We have a new $O(nC)$ approach!**
- ILP Models ⇒ Subject of this talk!
- CP Models
- Combinatorial lower bounds

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ILP FORMULATIONS

ILP Formulation 1

$$(\mathrm{ILP}_1) \qquad \min \sum_{i \in I} p_i x_i$$

$$\sum_{i \in I} w_i x_i \leq C \tag{0.1}$$

$$\sum_{j \in I, j \neq i} w_j x_j + w_i(1 - x_i) \geq (C + 1)(1 - x_i) \quad \forall i \in I : i \leq i_c \tag{0.2}$$

$$x_i \in \{0, 1\} \qquad \forall i \in I$$

Constraint (0.2) can be rewritten as

$$\sum_{j \in I, j \neq i} w_j x_j \geq (C_i + 1)(1 - x_i) \quad \forall i \in I : i \leq i_c.$$

where $C_i = C - w_i$.

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Strengthening Covering Inequalities

Let (for a fixed $i \in I$)

$$\tilde{C} = \begin{cases} C_i + 1, & i \leq i_c \\ C_c + 1, & \text{otherwise} \end{cases}$$

Proposition

For a given $i \in I$, coefficients of the associated covering inequalities

$$\sum_{j \in I, j \neq i} w_j x_j + (C_i + 1) x_i \geq C_i + 1$$

can be down-lifted to $\sum_{k \in I} \alpha_k x_k \geq \tilde{C}$ where

$$\alpha_k := \begin{cases} \min\{w_c, \tilde{C}\}, & k = i \\ \min\{w_k, \tilde{C}\}, & \textit{otherwise} \end{cases} \qquad \forall k \in I$$

Global covering constraint:

$$\sum_{j \in I} w_i x_i \geq C_c + 1 \quad (C_c = C - w_c.) \tag{0.3}$$

(ILP_1): Computations

- (ILP_1) Basic
- (ILP_1^l) Lifted Variant
- In both cases: Branch-and-Cut approach - capacity constraints are separated on the fly

ILP Formulation 2

$$(\mathrm{ILP}_2) \qquad \min \sum_{i \in I} p_i x_i$$

$$\sum_{i \in I} w_i x_i \leq C \tag{0.4}$$

$$\sum_{i \in I} w_i x_i + z \geq C + 1 \tag{0.5}$$

$$z \leq w_c - (w_c - w_i)(1 - x_i) \qquad \forall i \in I, i \leq i_c \tag{0.6}$$

$$x \in \{0, 1\}, z \in \mathbb{N} \qquad \forall i \in I$$

- a single additional variable, but significantly simplified structure.

Formulations' structures

Cap	9		x_1	x_2	x_3	x_4	x_5	z
p_i			1	1	1	3	8	
w_i			1	1	1	3	8	
sol			1	1	1	1	0	4
	OPT	LP						
ILP_1	6	4.0588	0.6176	0.6176	0.6176	0.7353	0.0000	
ILP_1^l	6	4.1803	0.6885	0.6885	0.6885	0.7049	0.0000	
ILP_2	6	3.9718	0.7183	0.7183	0.7183	0.6056	0.0000	6.0282

Model I

```
Minimize
 obj:   x1 +   x2 +   x3 + 3 x4 + 8 x5
Subject To
 c1:    x1 +   x2 +   x3 + 3 x4 + 8 x5 <= 9
 c2: 9 x1 +   x2 +   x3 + 3 x4 + 8 x5 >= 9
 c3:    x1 + 9 x2 +   x3 + 3 x4 + 8 x5 >= 9
 c4:    x1 +   x2 + 9 x3 + 3 x4 + 8 x5 >= 9
 c5:    x1 +   x2 +   x3 + 7 x4 + 8 x5 >= 7
 c6:    x1 +   x2 +   x3 + 3 x4 + 2 x5 >= 2
Binaries
        x1     x2     x3     x4     x5
End
```

Model II

```
Minimize
 obj:     x1 +    x2 +    x3 + 3 x4 + 8 x5
Subject To
 c1:      x1 +    x2 +    x3 + 3 x4 + 8 x5     <= 9
 c2:      x1 +    x2 +    x3 + 3 x4 + 8 x5 + z >= 10
 c3: - 7 x1                                + z <= 1
 c4:           - 7 x2                      + z <= 1
 c5:                  - 7 x3               + z <= 1
 c6:                         - 5 x4        + z <= 3
 c7:                                         z <= 8
Binaries
          x1      x2      x3      x4     x5
End
```

IS THERE A CONNECTION

BETWEEN ILP_1 and ILP_2?

A (not so) surprising result!

Proposition

Benderization of ILP_2:
Projecting out z variables from ILP_2 results in the formulation ILP_1 in which constraints (0.2) are down-lifted as follows:

$$\sum_{j \in I, j \neq i} w_j x_j + w_c x_i \geq C + 1 - w_i$$

Corollary

If $\min\{w_k, C + 1 - w_i\} = w_k$, for all $k, i \in I$, then the LP-relaxations of ILP_1^l and ILP_2 are the same.

Observation

If $\exists i, k$ s.t. $w_k + w_i > C + 1$ ILP_1^l can still be stronger than ILP_2: coefficients next to x_j, $j \neq i$ are not down-lifted!

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COMPUTATIONAL STUDY

Instances

- classical instance generator for 0/1 KP described in [6]
- two values: $\overline{R} \in \{1000, 10000\}$
- **small:** $n \in \{10, 20, 30, 40, 50, 100, 500, 1000, 2000\}$,
- **large:** $n \in \{7500, 10000, 15000, 20000\}$
- $C \in \{25\%, 50\%, 75\%\}$ of total weight
- 9 different groups, each with 54 instances $\Rightarrow$ 486 + 216 in total

Instances

1. Uncorrelated: w_j u.r. in $[1, \overline{R}]$, p_j u.r. in $[1, \overline{R}]$.
2. Weakly correlated: w_j u.r. in $[1, \overline{R}]$, p_j u.r. in $[w_j - \overline{R}/10, w_j + \overline{R}/10]$ so that $p_j \geq 1$.
3. Strongly correlated: w_j u.r. in $[1, \overline{R}]$, $p_j = w_j + \overline{R}/10$.
4. Inverse strongly correlated: p_j u.r. in $[1, \overline{R}]$, $w_j = p_j + \overline{R}/10$.
5. Almost strongly correlated: w_j u.r. in $[1, \overline{R}]$, p_j u.r. in $[w_j + \overline{R}/10 - \overline{R}/500, w_j + \overline{R}/10 + \overline{R}/500]$.
6. Subset-sum: w_j u.r. in $[1, \overline{R}]$, $p_j = w_j$.
7. Even-odd subset-sum: w_j even value u.r. in $[1, \overline{R}]$, $p_j = w_j$, c odd.
8. Even-odd strongly correlated: w_j even value u.r. in $[1, \overline{R}]$, $p_j = w_j + \overline{R}/10$, c odd.
9. Uncorrelated with similar weights: w_j u.r. in $[100\,\overline{R}, 100\,\overline{R} + \overline{R}/10]$, p_j u.r. in $[1, \overline{R}]$.

Greedy Heuristics

Table : Average percentage gaps from best known solutions by greedy heuristics.

	Class									
Algorithm	1	2	3	4	5	6	7	8	9	avg
Greedy heuristics										
greedy[$1/p_j$]	29.25	7.53	11.83	1.55	11.78	2.23	2.24	11.76	2.94	9.01
greedy[$1/w_j$]	66.85	9.08	11.83	1.55	11.78	2.23	2.24	11.76	55.35	19.19
greedy[w_j/p_j]	6.71	2.16	1.92	1.55	2.11	2.23	2.24	1.85	2.94	2.63
greedy[$1/(p_j * w_j)$]	56.20	8.35	11.83	1.55	11.78	2.23	2.24	11.76	2.94	12.10
greedy[$1/(p_j + w_j)$]	56.35	8.39	11.83	1.55	11.78	2.23	2.24	11.76	4.20	12.26
greedy[p_j/w_j]	71.34	19.67	11.83	22.03	11.78	2.23	2.24	11.76	68.07	24.55
greedy-comb	6.71	1.14	1.03	1.55	1.07	2.23	2.24	0.96	2.82	2.19

- Observe: sorting according to w_j/p_j performs the best, on average, which is also intuitive (we prefer items with low profit and high weight).

Formulation relative strength

Table : Average percentage gaps of the different linear programming relaxations.

	Class									
Algorithm	1	2	3	4	5	6	7	8	9	avg
Linear Relaxation										
ILP_1	20.11	41.52	40.09	45.62	39.90	45.35	45.35	40.17	22.98	37.90
ILP_1^l	13.67	3.19	2.75	2.52	2.68	2.92	2.93	2.84	6.64	4.46
ILP_2	13.67	3.19	2.79	2.52	2.68	2.95	2.96	2.88	6.64	4.47

- Lifting significantly reduces the LP-gap of (ILP_1)
- (ILP_2) provides very strong lower bounds, comparable to those obtained after lifting (ILP_1).
- The first class of instances (uncorrelated weights and profits) is by far the most difficult one, with average LP-gaps of more than 13% for all ILP formulations.

Formulation Comparison: Over all classes (**small**)

Items	ILP_1	ILP_1 ILP'_1	B&C	ILP_2	DP
Avg *t***[sec.s]**					
10	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0
30	0.2	0.1	0.1	0.1	0.0
40	0.4	0.2	0.2	0.2	0.1
50	0.9	0.2	0.7	0.3	0.1
100	34.7	2.9	19.5	1.4	0.3
500	32.1	8.8	70.1	0.5	1.3
1000	150.7	18.8	32.1	1.1	5.2
2000	105.1	73.8	107.0	7.5	8.9
AVG	19.9	10.5	14.2	1.2	1.7
# of TL					
10	0	0	0	0	0
20	0	0	0	0	0
30	0	0	0	0	0
40	0	0	0	0	0
50	0	0	0	0	0
100	2	0	1	0	0
500	24	4	27	5	3
1000	28	2	32	3	3
2000	44	9	36	5	5

Formulation Comparison

Table : Class 1-2-3

	Avg t[sec.s]			# of TL		
Items	ILP_1^l	ILP_2	DP	ILP_1^l	ILP_2	DP
7500		16.8	105.7	10	0	0
10000		38.8	191.2	10	0	0
15000		16.0	417.6	10	0	0
20000		58.4	737.5	10	0	0
7500		13.3	118.6	10	0	0
10000		28.6	206.7	10	0	0
15000		28.5	437.2	10	0	0
20000		2.3	700.1	10	0	0
7500	27.4	3.2	106.2	3	0	0
10000		7.7	171.1	10	0	0
15000		28.6	407.2	10	0	0
20000		5.3	658.5	10	0	0

Formulation Comparison

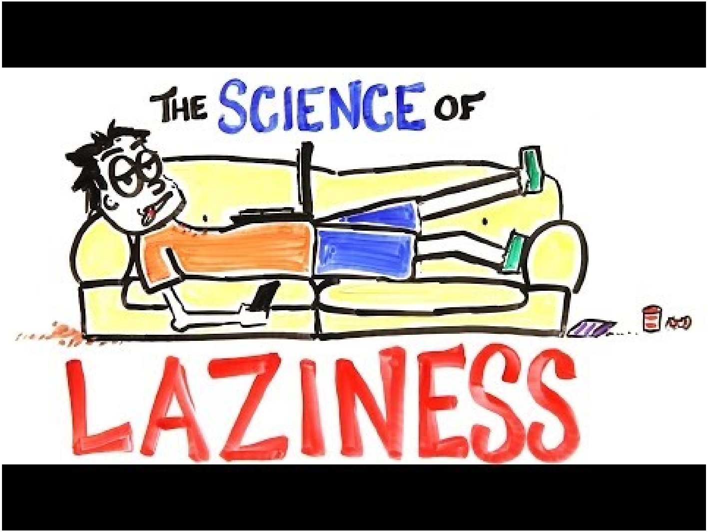
Table : Class 4-5-6

	Avg t[sec.s]			# of TL		
Items	ILP_1^l	ILP_2	DP	ILP_1^l	ILP_2	DP
7500	15.1	8.0	131.0	3	2	0
10000		2.9	238.1	10	2	0
15000		3.1	503.7	10	0	0
20000		15.1	643.3	10	1	1
7500		115.1	107.8	10	4	0
10000		127.1	192.8	10	2	0
15000		5.7	419.4	10	3	0
20000		3.8	701.6	10	2	0
7500		4.3	104.9	10	0	0
10000		4.4	183.4	10	1	0
15000		9.2	393.7	10	0	0
20000		59.6	687.3	10	0	0

Formulation Comparison

Table : Class 7-8-9

	Avg t[sec.s]			# of TL		
Items	ILP_1^l	ILP_2	DP	ILP_1^l	ILP_2	DP
7500		20.7	104.0	10	0	0
10000		5.5	185.1	10	0	0
15000		20.2	400.2	10	0	0
20000		15.6	679.2	10	0	0
7500	34.6	0.9	103.5	3	0	0
10000		2.6	198.6	10	0	0
15000		6.7	396.9	10	0	0
20000		2.3	683.9	10	0	0
7500	18.0	3.7	746.6	4	1	7
10000	32.6	0.3		2	3	10
15000		0.6		10	1	10
20000		0.7		10	6	10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Recap and future developments

- We studied ILP and DP approaches for solving the LBP problem to optimality. ILP_1^l can be obtained by "Benderization" of ILP_2
- Our computational study showed that the LBP is more difficult than the KP:
  - for the latter, instances with several thousands of items can be easily solved,
  - while for the LBP, a few thousands items make the problem difficult.
- **Greedy Boss:** Find a most profitable subset of jobs S^* to be executed so that the schedule exceeds the deadline, but after removing any of the scheduled jobs, the deadline is respected.
  - solution properties, valid inequalities, solution approaches
- **Lazy Bureaucrat vs Greedy Boss Games!**

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